1) $|P_\varphi| = \sqrt{\ell(\ell+1)}\ \hbar$

$\ell = 4$

$= \sqrt{20}\ \hbar$

2) $m_\ell = 1$

z- Component $= 1\hbar$

3) $n = 5$

$E = -13.6/n^2$ e.V.

$= -\frac{13.6}{5^2} = -0.54$ eV

Consider An Example for the H-atom

Given: $\psi_{n,l,m_l} = \psi_{5,4,1}$

1) Magnitude of the Orbital Angular Momentum?

2) What is the z-Component of the orbital angular momentum?

3) What is the value of the energy?

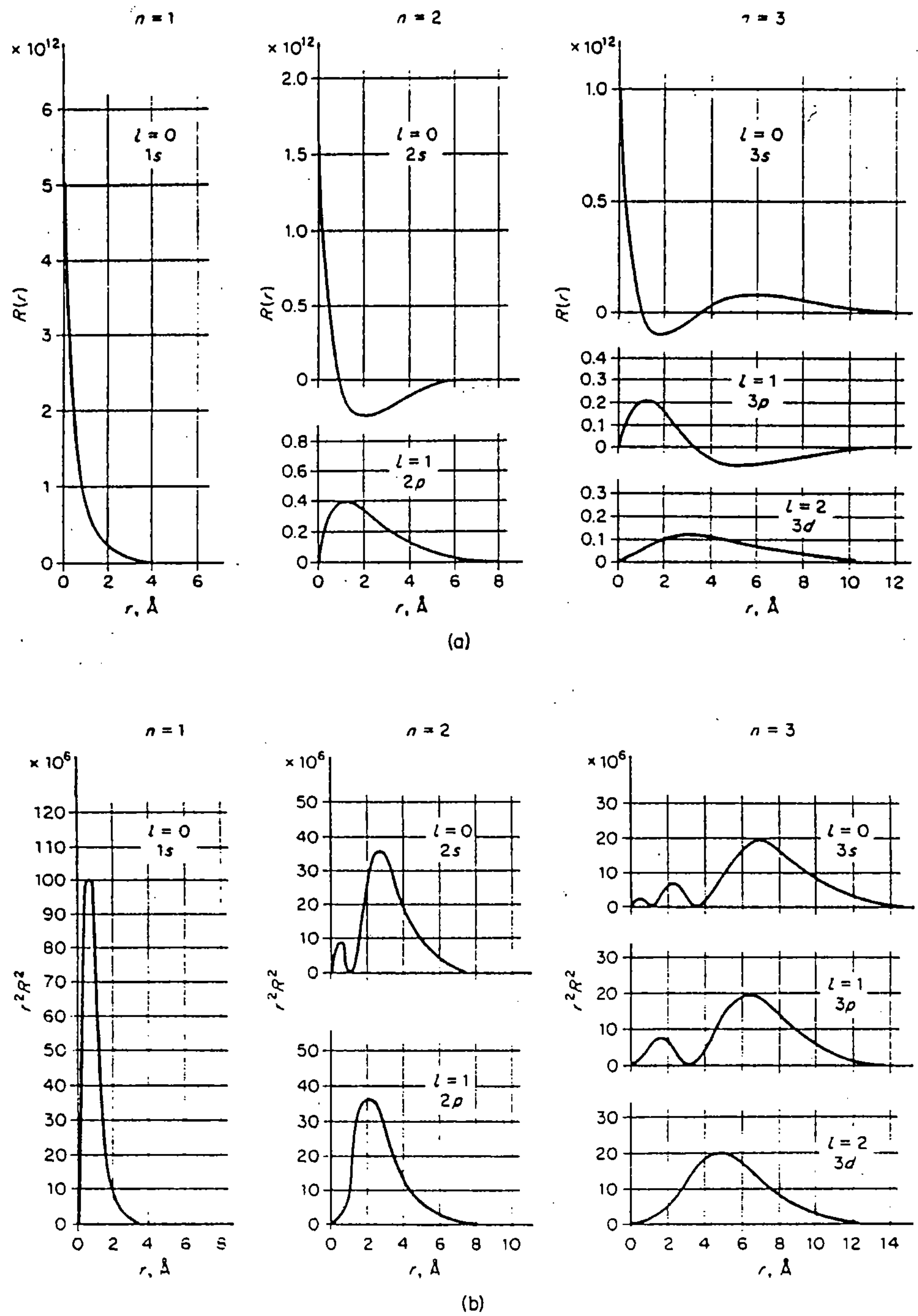
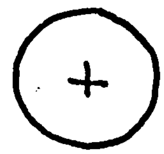


FIG. 12.18 (a) Radial part of wave functions for hydrogen atom; (b) Radial distribution functions — giving probability of finding electrons at a given distance from nucleus. [After G. Herzberg, *Atomic Spectra* (New York, Dover, 1944)].

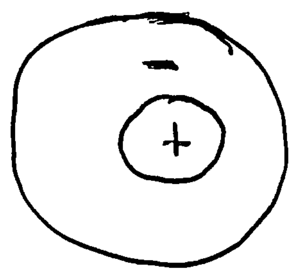
Consider the Nodal Properties of ψ

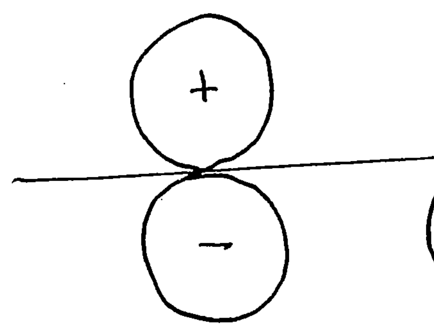
n = 1

1s  no nodes

---

n = 2

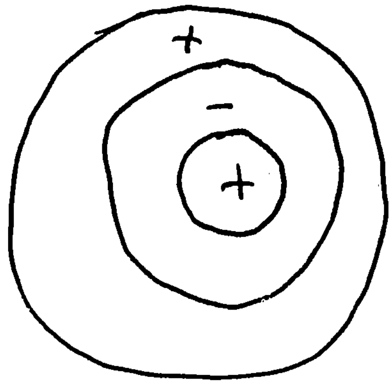
2S  1 node (radial)

2P  1 node (angular planar.

Radial Nodes (continued)

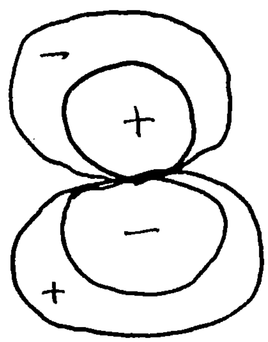
n = 3

3s



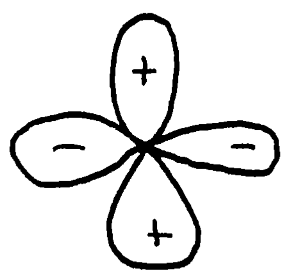
2 nodes
(radial)

3p



2 nodes
(1 radial
1 planar)

3d



2 nodes
(angular, planar)

Rule: For H- like systems:

ALL states at a given energy level have the same number of nodes = n − 1

General Rules for the Angular & Radial Nodes

Total # of nodes $= n-1$

# of angular nodes $= \ell$

$\therefore$ number of radial nodes $= n - \ell - 1$

n	ℓ	# Nodes: angular	radial	Total
1	0	0	0	0
2	0	0	1	1
2	1	1	0	1
3	0	0	2	2
3	1	1	1	2
3	2	2	0	2

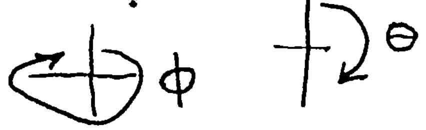
Consider the Probability of Finding an Electron at a Given Location

$$\int P(r)\,dr = \int |\psi|^2 \, d\tau \Leftarrow d\,\text{Volume}$$

The volume element in 3-d, polar coordinates

$$d\tau = r^2 dr \sin\theta \, d\theta \, d\phi$$

$$P(r)\,dr = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} |\psi|^2 r^2 dr \sin\theta \, d\theta \, d\phi$$



$$\int_0^{2\pi} \int_0^{\pi} Y_{\ell, m_\ell}(\theta,\phi) \sin\theta \, d\theta \, d\phi = 1$$

$$P(r)\,dr = 4\pi r^2 |R|^2 dr$$

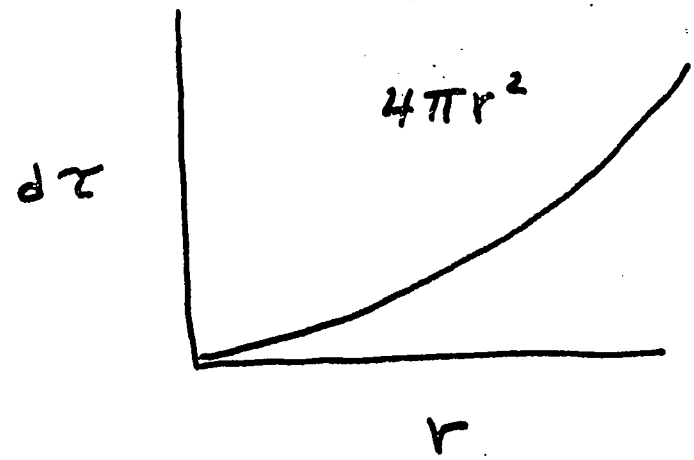
Next Page

Consider

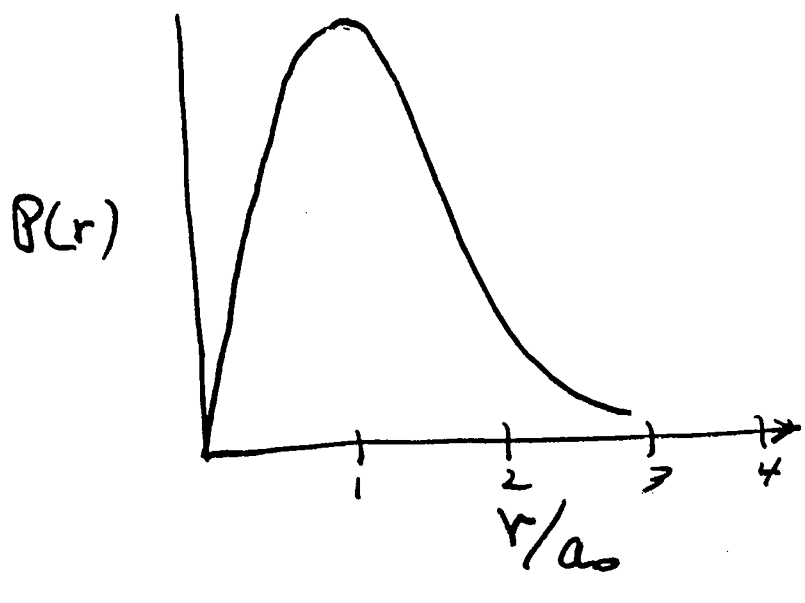
$$\int\int\underbrace{\int r^2 dr}\ \sin\theta\, d\theta\, d\phi$$

$$\underbrace{\underbrace{\int_0^{2\pi} d\phi}_{2\pi}\ \underbrace{\int_0^{\pi} \sin\theta\, d\theta}_{2}}_{4\pi r^2 dr}\ r^2 dr$$

Consider 1s Electrons



product



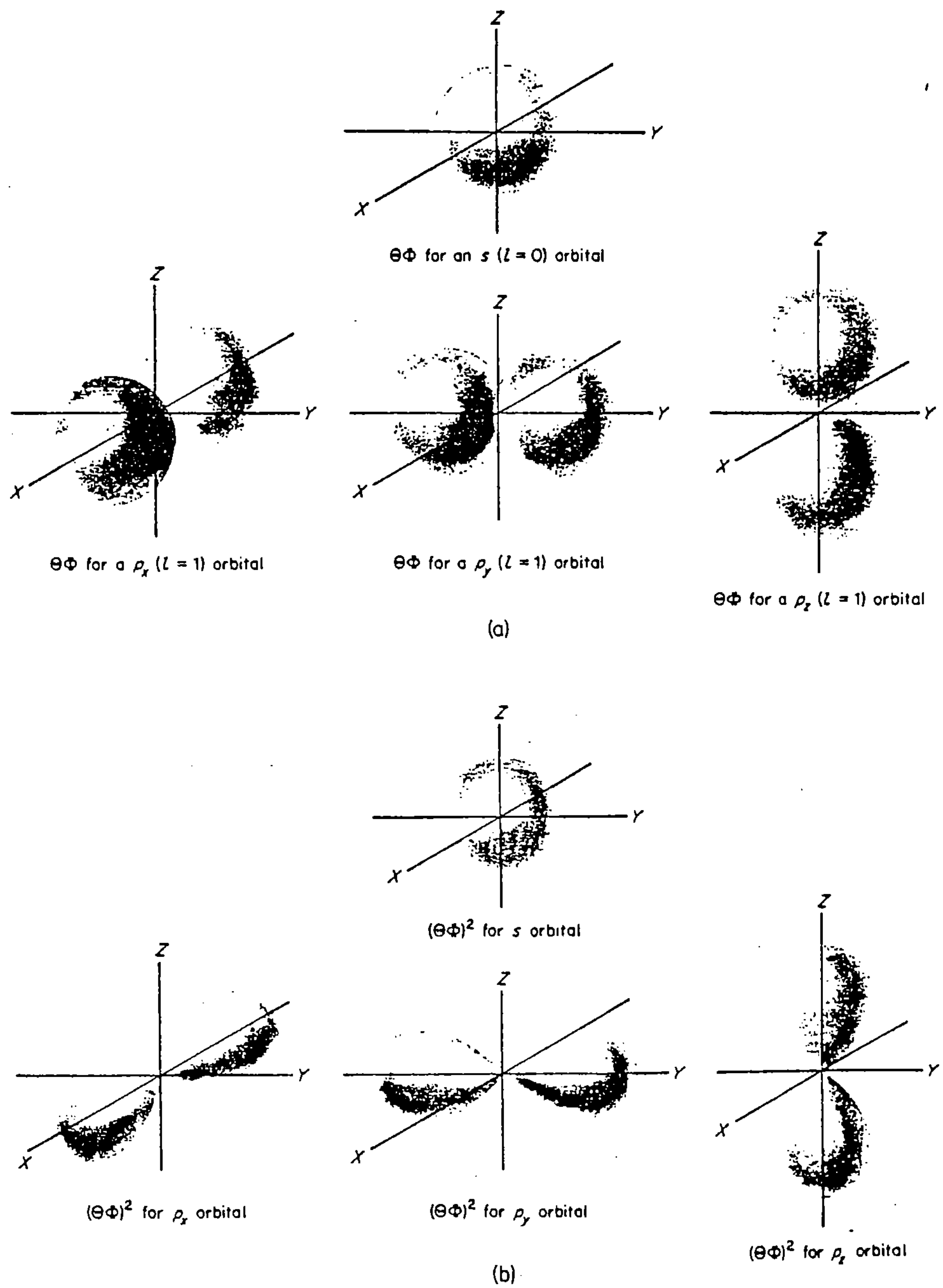
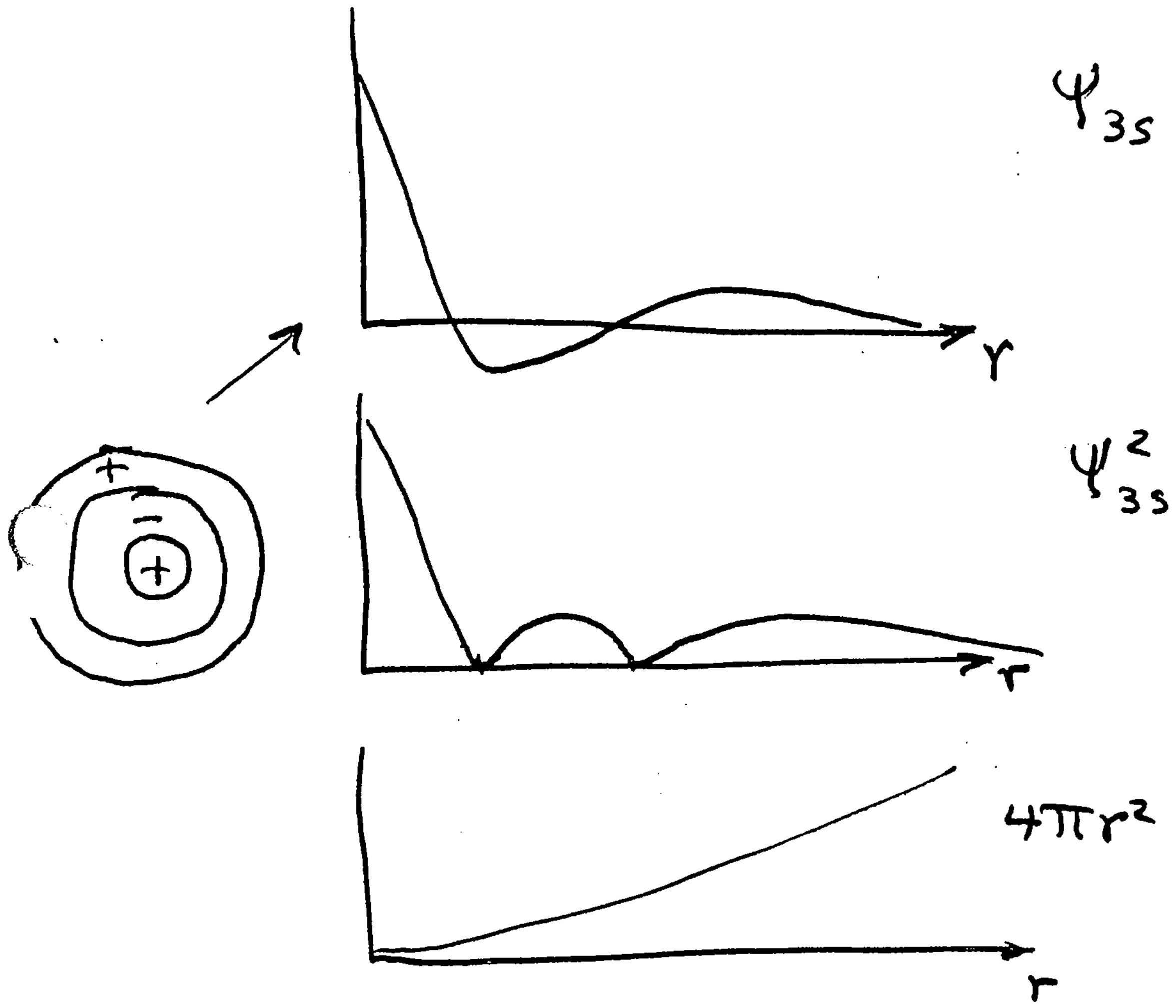


FIG. 12.19 Polar graphs: (a) the absolute values of ΘΦ, the angular part of the hydrogen-atom wave functions for $l = 0(s)$ and $l = 1(p)$ orbitals; (b) $(\Theta\Phi)^2$ which is proportional to the electron densities.

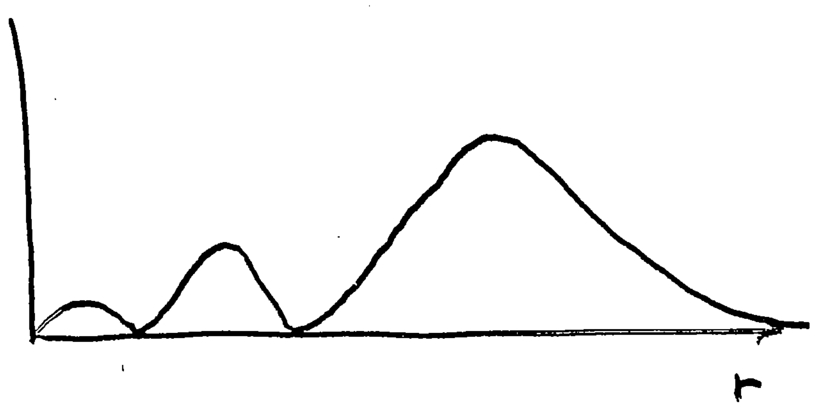
Consider 3s Electrons

$n = 3 \qquad \ell = 0$

$\therefore$ 2 radial nodes



$4\pi r^2 \psi^2_{3s}$



most electronic charge lies outside the 2nd radial node

The General Idea of Selection Rules

$\Delta E = h\nu$ frequency of a given transition given by energy difference between two levels

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Photon: Possesses intrinsic spin angular momentum corresponding to $s = 1$

Suppose atom in d-orbital:

then a photon emitted:

System cannot drop to s-orbital

$$l = 2 \not\Rightarrow l = 0$$

Likewise: $s \not\Rightarrow s$

not allowed

Some transitions allowed / some ~~for~~ forbidden

Selection Rules : tell us what is allowed.

Case : 1-electron atom

$$\Delta n = \text{any integer}$$

$$\Delta \ell = \pm 1$$

$$\left[\Delta m_\ell = 0 \text{ or } \pm 1 \right]$$