Heisenberg's Uncertainty Principle

- The very act of observing affects the particle being observed.
- Both momentum and position can't be known with infinite precision

Consider: particle in a 1-D box.

Desire: measure momentum component in the x-direction for a set of identical systems for which the particle is known to be in the lowest energy state.

i.e. $n = 1$ $\quad \psi = A \sin \frac{n\pi x}{L}$

$$\hat{P}_x = -i\hbar \frac{d}{dx}$$

Consider: $\hat{P}_x \psi$

$$\hat{P}_x \psi = -i\hbar \frac{d}{dx}\left(A \sin \frac{\pi x}{L}\right)$$

$$= -i\hbar A \frac{\pi}{L} \cos \frac{\pi x}{L}$$

operating on $\sin \Rightarrow \cos$

$\therefore$ - ψ not an eigenfunction of $\hat{P}$

- measurements will not yield a precise result.

Need to use the mean value theorem to obtain the expectation value.

$$\langle P_x \rangle = \int_0^L \psi_1 \underbrace{\hat{P}_x \psi_1}_{-i\hbar A \frac{\pi}{L} \cos \frac{\pi x}{L}} dx$$

$$A = \left(\frac{2}{L}\right)^{1/2}$$

$$\langle P_x \rangle = \left(\frac{2}{L}\right)^{1/2} \left(\frac{2}{L}\right)^{1/2} \left(-i\hbar \frac{\pi}{L}\right) \int_0^L \sin \frac{\pi x}{L} \cos \frac{\pi x}{L} dx$$

$$= 0$$

propagation in both directions

Suppose

$$\hat{O} = \left(-i\hbar \frac{d}{dx}\right)\left(-i\hbar \frac{d}{dx}\right)$$

$$= -\hbar^2 \frac{d^2}{dx^2} = P_x^2$$

$$\hat{O} = -\hbar^2 \frac{d^2}{dx^2}\left[\left(\frac{2}{L}\right)^{1/2} \underbrace{\sin \frac{\pi x}{L}}\right]$$

$$= \hbar^2 \left(\frac{2}{L}\right)^{1/2} \frac{\pi^2}{L^2} \cdot \sin \frac{\pi x}{L}$$

Ψ

$\therefore$ P_x^2 is an eigenvalue

A series of measurements on identical systems will always give a precise result.

$$\therefore \quad P_x^2(n=1) = \frac{\hbar^2 \pi^2}{L^2} = \frac{h^2}{4L^2}$$

But **1-D Particle in a Box**

$$E_n = \frac{n^2h^2}{8mL^2} \Rightarrow \frac{h^2}{8mL^2} \quad (n=1)$$

$$P_x^{\,2} = \frac{h^2}{4L^2}$$

Hence $P_x^{\,2} = 2mE_1 \quad (\leftarrow n=1)$

$$P_x^{(1)} = \pm\sqrt{2mE_1}$$

Ave. Value $P_x^{(1)} = 0$

Always some +, some −

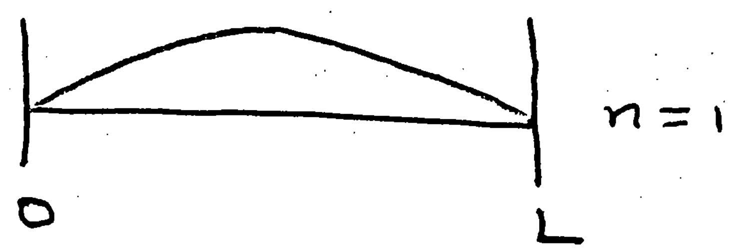
Never know which you will measure

An Uncertainty exists in ones Knowledge $= 2(2mE_1)^{1/2}$

Momentum

Now, Consider Position

$\psi_n \Rightarrow \psi_1$ = State of the particle



Uncertainty in position — particle is somewhere in the box

$$\Delta P_x \, \Delta X \geq 2(2mE_1)^{1/2} \cdot L$$

product of the uncertainties

$$\geq 2\left(2m \frac{h^2}{8mL^2}\right)^{1/2} \cdot L$$

$$\geq h$$

More Exact Reasoning $\quad dPdq \geq \hbar/2$