The possible measured values of a property of a system are the eigenvalues of the operator associated with that property.

When the wavefunction of a particle is <u>Not</u> an eigenfunction of an operator, the property is indefinite,

Considering Eigenvalues / Eigenfunctions

Consider $\hat{O}$ = operator

f = function

let $f = e^{ax}$ $\quad \hat{O} = \frac{d}{dx}$

$$\frac{d}{dx}\left(e^{ax}\right) = a\,e^{ax}$$

$$\therefore \ \hat{O} f = a f$$

a = eigenvalue

e^{ax} is eigenfunction of the operator $\frac{d}{dx}$

Observables

$$\hat{O}\,\Psi = O\,\Psi$$

operator $\quad$ observable

- If know Ψ
- If know operator corresponding to an observable of interest

$\therefore$ Can determine the outcome of an observation of that property

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Postulate of Quantum Mechanics:

Every observable property of a system is associated with an operator

How to find? Write classical expression for operator in terms of $x, y, z \cdots P_x, P_y, P_z$

Replace P_x, P_y, P_z with $\frac{\hbar}{i}\frac{d}{dx}, \frac{\hbar}{i}\frac{d}{dy}, \frac{\hbar}{i}\frac{d}{dz}$

Consider the Free Particle

$$\psi = \underline{\underline{A}}\ \underline{\underline{e}}^{ikx}$$

$$\hat{p} = \left(\frac{\hbar}{i}\right)\frac{d}{dx}$$

the linear momentum operator in 1-D

$$\hat{p}\,\psi = p\,\psi$$

$$\frac{\hbar}{i}\frac{d\psi}{dx} = p\,\psi$$

$$\frac{\hbar}{i}\frac{d}{dx}\left(A e^{ikx}\right) = p\left(A e^{ikx}\right)$$

$$\frac{\hbar}{\cancel{i}}\,\cancel{A}\,\cancel{i}k\,\cancel{e^{ikx}} = p\,\cancel{A}\,\cancel{e^{ikx}}$$

$$\hbar k = p = \frac{h}{2\pi}\,\frac{2\pi}{\lambda} = h/\lambda$$

If $\psi = B e^{-ikx}$

$$p = -\hbar k$$

(moving opposite direction)

Consider the Free Particle

Apply the superposition principle

$$\Psi = A e^{ikx} + A e^{-ikx}$$

gave $p = \hbar k$ $\quad$ $p = -\hbar k$

$$\Psi = A\left(e^{ikx} + e^{-ikx}\right) = 2A \cos kx$$

$$\hat{p} = \frac{\hbar}{i}\frac{d}{dx}$$

$$\therefore \hat{p} = -\frac{\hbar}{i} 2A k \sin kx$$

Since not an Eigenfunction, Q.M. says: this is taken to mean p is indefinite

In this case, not <u>completely</u> true:

Cosine linear superposition of e^{ikx} and e^{-ikx} $\leftarrow$ but both Eigenfunctions $\hat{p}$ each $|k\hbar|$

$\therefore$ half measurements show particle moving to right, ½ to left

General Case of Superposition

Consider a Ψ for momentum

$$\Psi = c_1 \Psi_{\text{momentum } 1} + c_2 \Psi_{\text{momentum } 2} + \cdots$$

Means?

- If corresponding eigenfunction occurs in the superposition — one of the values of momentum will be measured — which one is not predictable
- In a series of measurements, the probability of measuring momentum i is proportional to $c_i^2 \rightarrow (c_i^* c_i)$

Average Value = Expectation Value

$$\langle O \rangle = \int \psi^* \hat{O} \psi \, d\tau$$

The above is correct for normalized wave functions.

Otherwise:

$$\int_{\text{all space}} (N\psi^*)(N\psi)\, d\tau = 1$$

$$N = \left[\frac{1}{\int \psi^* \psi \, d\tau} \right]^{1/2}$$

$$\langle O \rangle = \frac{\int \psi^* \hat{O} \psi \, d\tau}{\int \psi^* \psi \, d\tau}$$

Mean Value Theorem

Mean Value Theorem

Tells us what the experimental result will be when a system is Not described by an eigenfunction of the operator involved.

But, Iff ψ is an eigenfunction of $\hat{O}$, the average value will be the same as the eigenvalue