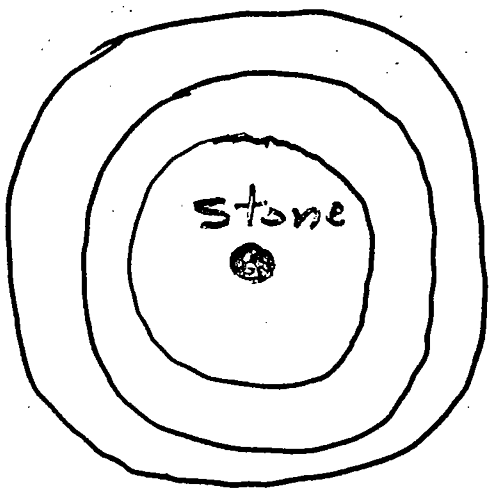
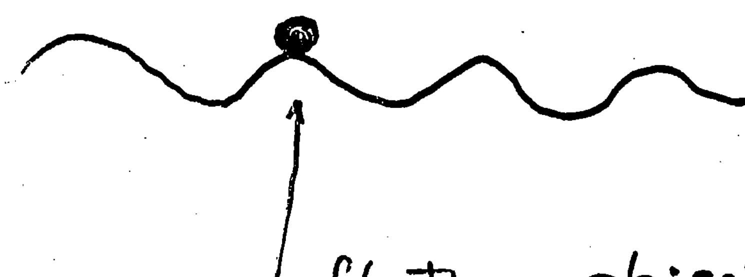
Consider Waves



traveling waves

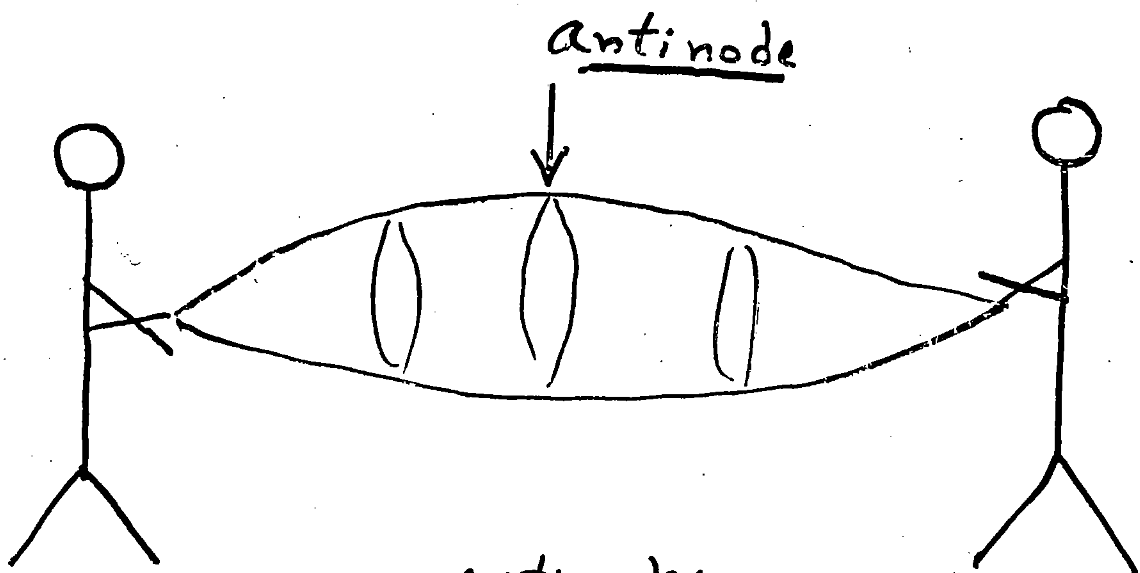
- transverse
- longitudinal

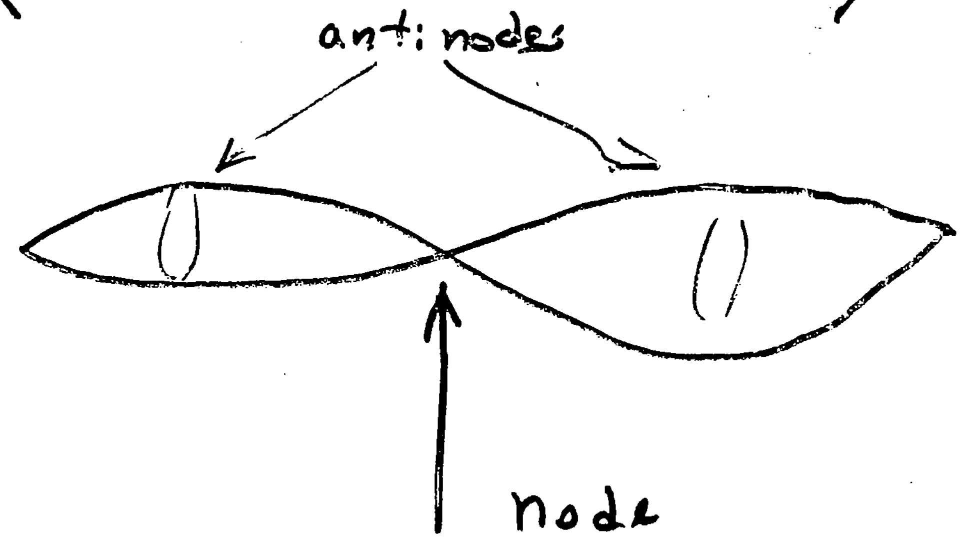


floating object is not translated very much

Standing Waves

no net forward motion





jump rope ≡ transverse

air/sound ≡ longitudinal

Waves – Consider movement in a single direction

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v = velocity
t = time
x = position

after time t, wave profile the same as original.

refer disturbance back to stationary origin:

$$x - vt$$

Convenient to scale with distance ~ dimension

$\lambda \equiv$ wavelength = distance over which wave repeats itself

Scaling the wave

$$\frac{1}{\lambda}(x - vt) = \frac{x}{\lambda} - \underbrace{\frac{v}{\lambda}}_{\nu = \frac{v}{\lambda}} t$$

# of replacas that pass a given point per unit time

$\therefore$ frequency

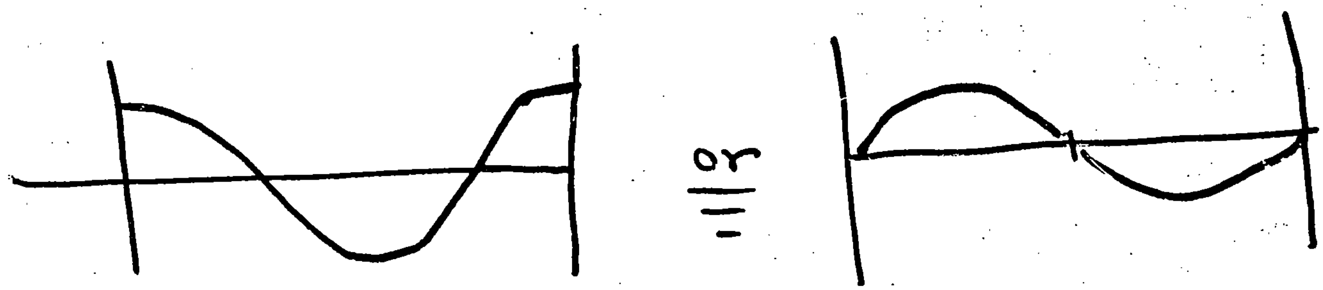
$$\bar{\nu} = \frac{1}{\lambda} \equiv \text{Wave number}$$

# of wavelengths in unit distance

Waves repeat in 2π

$\therefore$ Convenient to describe wave in terms of some equivalent circular motion

Similar to **harmonic Osc.**



Cos Sin

$$k\,(x - vt) = \frac{2\pi}{\lambda}\,(x - vt)$$

ω = angular frequency

$= 2\pi\nu \qquad \nu = \dfrac{v}{\lambda}$

$\rightarrow kx - \omega t$

$$f(x,t) = A \sin(kx - \omega t + \varphi)$$

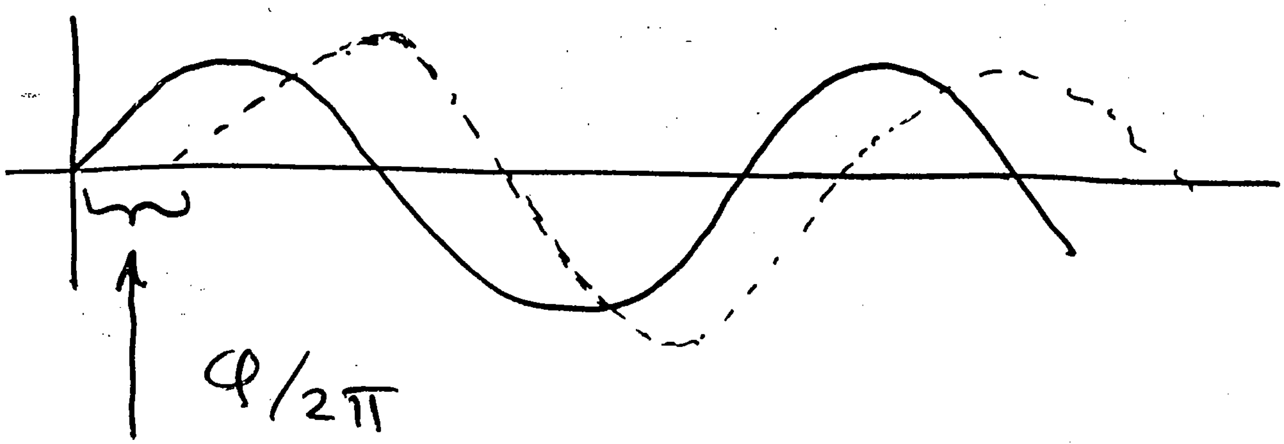
phase angle

Stationary / Standing Waves

Consider ϕ_1 and ϕ_2 have the same amplitude

$$\phi_1 = A \sin 2\pi(\bar{\nu}x - \nu t)$$

$$\phi_2 = A \sin\left[2\pi(\bar{\nu}x - \nu t) - \varphi\right]$$



If: $\varphi = 2n\pi$ ∴ even multiple,
then ⇒ two in phase

$\varphi = (2n+1)\pi$ ∴ odd multiple
out of phase
then ⇒ interfere & sum to zero

Partial Differential Equation of Wave Motion:

$\phi \Rightarrow$ disturbance:

1-D $$\frac{\partial^2 \phi}{\partial x^2} = \frac{1}{V^2} \frac{\partial^2 \phi}{\partial t^2}$$

3-D $$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \phi = \frac{1}{V^2} \frac{\partial^2 \phi}{\partial t^2}$$

— all terms of 1st degree

— coefficients are constants

Superposition principle

if ϕ_1 ; ϕ_2 are solutions

then $a_1 \phi_1 + a_2 \phi_2$ is a solution

$\therefore$ Can sum solutions to obtain other solutions

Consider two identical waves moving in opposite directions

$$\phi = A \sin 2\pi(\tilde{\nu}x - \nu t) + A \sin 2\pi(\tilde{\nu}x + \nu t$$

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\phi = 2A \sin 2\pi\tilde{\nu}x \cos 2\pi\nu t$$

$\therefore$ irrespective of t, get nodes:

$$x = 0, \frac{1}{2\tilde{\nu}}, \frac{1}{\tilde{\nu}}, \frac{3}{2\tilde{\nu}}, \cdots \frac{n}{2\tilde{\nu}}$$

Consider a rope of length a

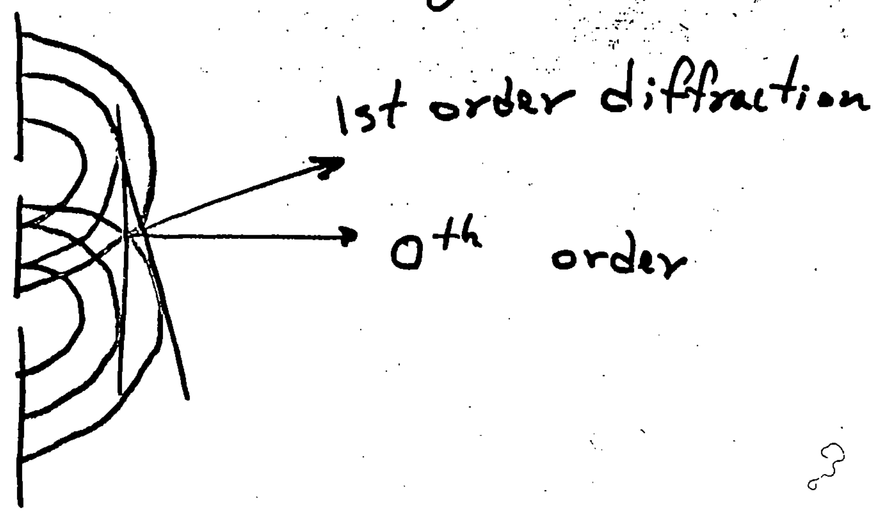
integral # of nodes between 0 and a and at 0 and a

to prevent destruction by interference

$$n\frac{\lambda}{2} = a$$

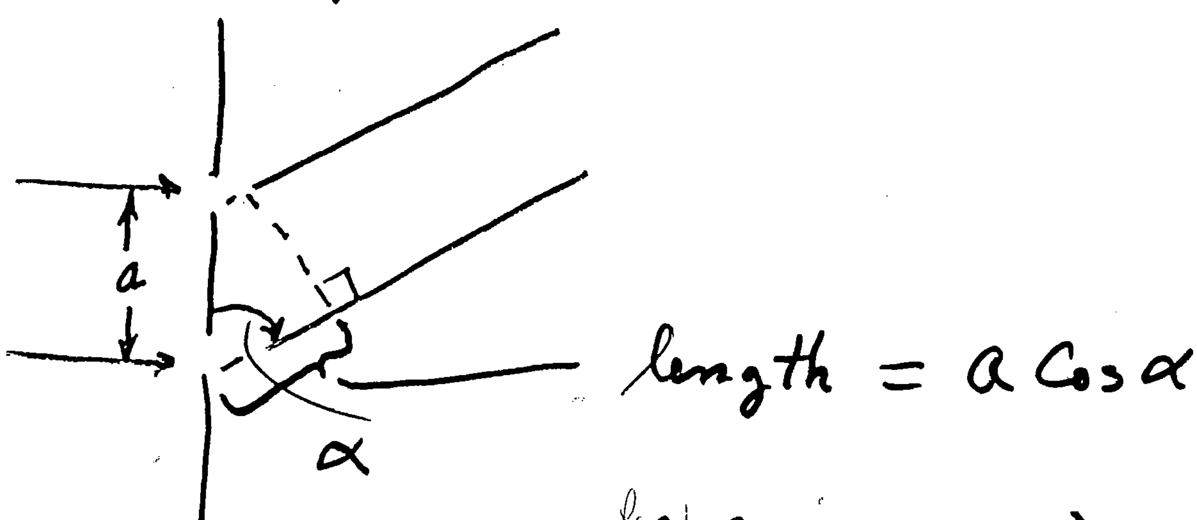
$$n = \text{integer}$$

Huygens – Consider plane wave front from a single source – incident on set of slits – Each slit is a new light source



two diffracted rays – reinforce in phase

requires difference in path for the two rays to be integral # of wave lengths



$n\lambda \; a \cos\alpha = n\lambda$